# Financial Markets I

**Lecture 6: The CAPM**

**Master Finance & Strategy**

**Spring 2018**

SciencesPo

# General Overview

- In Lecture 5 (*Portfolio Theory*), we studied how to choose a stock portfolio.
- In this lecture, we seek to obtain some insight on stocks' expected returns.
  - How are expected returns determined?
  - How are they related to risk?
  - What is the relevant measure of risk?
- When put in a demand/supply equilibrium context, portfolio theory delivers neat insights on expected returns and their relation to risk.

# Overview of Lecture 6

1. Linear Regressions: Basic Notions.
2. Regression and Asset Returns.
3. The CAPM.
4. The CAPM's Basic Insight.
5. CML vs. SML.
6. Uses of the CAPM.

# 1. Linear Regressions

- Suppose we have observations for two variables $X$ and $Y$, e.g.,
  - annual returns $R_{1,t}$ and $R_{2,t}$ on two stocks over time $t = 1, \ldots, T$
  - firm characteristic $X_i$ and average return $\bar{R}_i$ over stocks $i = 1, \ldots, N$.

- We can always describe the relationship between $X$ and $Y$ via a linear regression model of the form

$$Y = a + bX + \epsilon,$$

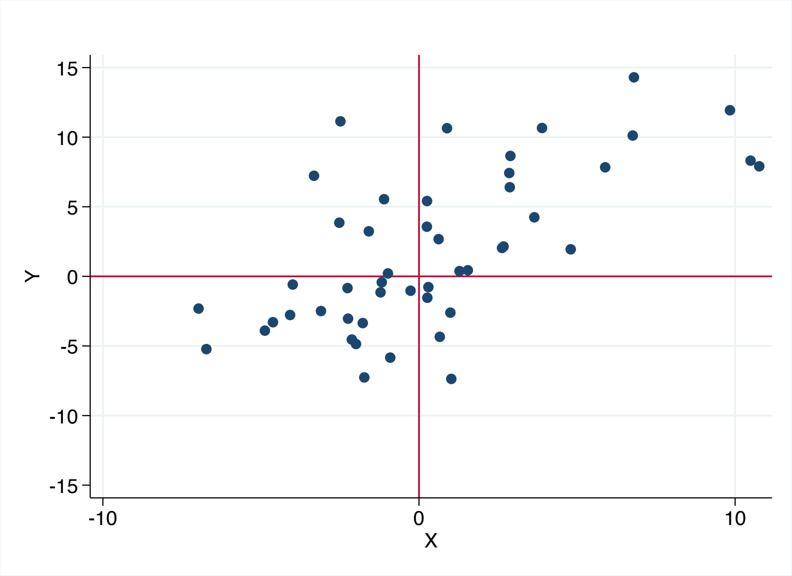
where the residual $\epsilon$ is such that $\bar{\epsilon} = 0$ and $Cov(X, \epsilon) = 0$.

- Interpretation: the variation in $Y$ is decomposed into
  - $bX$: variation in $Y$ that is "explained" by $X$;
  - $\epsilon$: variation in $Y$ that is not "explained" by $X$.

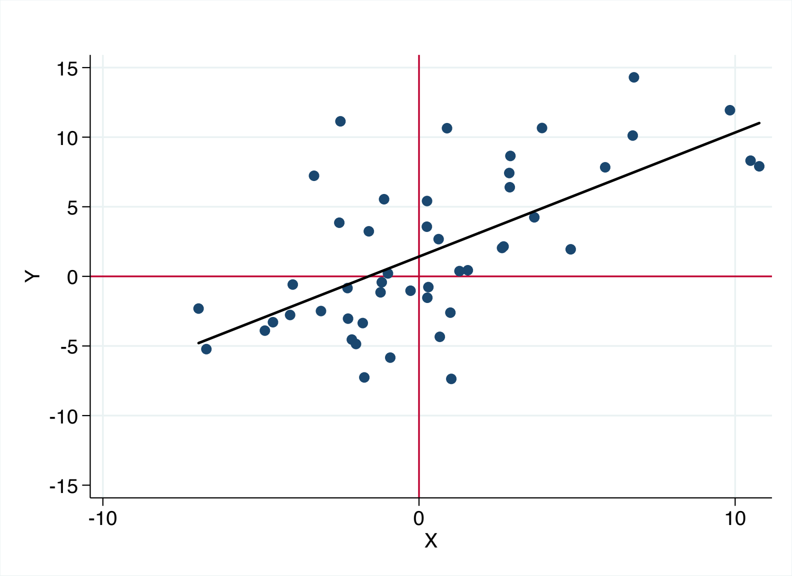
# Regression Estimation

- For a given sample of observations for $X$ and $Y$, the coefficients $a$ and $b$ can be estimated by regression analysis.
- Regression analysis:
  - We consider the scatterplot of $Y$ vs. $X$ (as shown next slide).
  - We fit the "best" line to the scatterplot.
  - Say that line is
  $$y = a + bx.$$
  - Then $a$ is the estimate for the 'intercept' coefficient, and $b$ is the estimate for the 'slope' coefficient.
- This gives us a *statistical* description of the dependence of $Y$ on $X$.
  - Caveat: in general, it should not be interpreted in terms of 'causality'.

# Scatterplot



# Scatterplot & Regression Line



# Regression Estimates

- Fact 1: the slope coefficient estimate is given by

$$b = \frac{Cov(X, Y)}{V(X)}.$$

- Fact 2: The intercept coefficient is given by

$$a = \bar{Y} - b\bar{X}.$$

- The residual is simply $\epsilon = Y - (a + bX)$.

- The estimation results depend on the sample statistics $\bar{X}$, $\bar{Y}$, $V(X)$, and $Cov(X, Y)$, and therefore are sensitive to the particular data sample we are using for estimation.

# Regression Output

Any regression package gives a number of outputs:

- Estimate for intercept coefficient, $a$.
  - Standard error of estimate, $s_a$ (measure of estimation uncertainty).
- Estimate for slope coefficient, $b$.
  - Standard error of estimate, $s_b$ (measure of estimation uncertainty).
- Standard deviation of residual $\epsilon$, $s(\epsilon)$.
- R-Square gives fraction of the variation in $Y$ "explained" by $X$.

$$\text{R-Square} = \frac{V(bX)}{V(Y)} = \frac{V(bX)}{V(bX) + s(\epsilon)^2}.$$

Note: $V(Y) = V(a + bX + \epsilon) = V(bX) + V(\epsilon)$, since $Cov(X, \epsilon) = 0$.

# 2. Regression and Asset Returns

- Suppose we have historical return data for asset $n$ (e.g., a stock) and the market (e.g., S&P500 index).
  - i.e., we observe $R_{n,t}$ and $R_{M,t}$ for $t = 1, \ldots, T$.

- Given the riskfree rate $R_f$, we can compute **excess returns:**
  - excess return on asset $n$, $R_n - R_f$,
  - excess return on the market, $R_M - R_f$.

- We can regress the excess returns of asset $n$ on the market excess returns. In this case,
  - $Y$ is excess return of asset $n$,
  - $X$ is excess return on the market.

# Regression and Asset Returns

- Regression equation for asset $n$ is

$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \epsilon_n.$$

- This provides a decomposition of the 'variation' in asset $n$ returns into two components:
  - $\beta_n(R_M - R_f)$ captures **systematic risk**, i.e., risk that is perfectly correlated with the 'aggregate' market.
  - $\epsilon_n$ captures **idiosyncratic risk**, i.e., risk that is uncorrelated with the market. By construction, $Cov(R_M - R_f, \epsilon_n) = 0$.
- Any asset is then characterized by its alpha, beta, and 'sigma'.
  - 'sigma' is the standard deviation of the idiosyncratic component, $s(\epsilon)$.

# Regression Example

IBM vs. S&P500 (market portfolio). Monthly returns Jan 1982-Dec 2017.

| | date | IBM exc ret | SP500 exc ret |
|---|---|---|---|
| 1 | date | IBM exc ret | SP500 exc ret |
| 314 | 31/01/08 | -1,16 | -6,36 |
| 315 | 29/02/08 | 6,54 | -3,61 |
| 316 | 31/03/08 | 0,94 | -0,78 |
| 317 | 30/04/08 | 4,73 | 4,65 |
| 318 | 30/05/08 | 7,55 | 0,97 |
| 319 | 30/06/08 | -8,60 | -8,77 |
| 320 | 31/07/08 | 7,83 | -1,13 |
| 321 | 29/08/08 | -4,62 | 1,09 |
| 322 | 30/09/08 | -4,07 | -9,23 |
| 323 | 31/10/08 | -20,59 | -17,02 |
| 324 | 28/11/08 | -11,71 | -7,51 |
| 325 | 31/12/08 | 3,13 | 0,78 |
| 326 | 30/01/09 | 8,90 | -8,57 |
| 327 | 27/02/09 | 0,95 | -11,01 |
| 328 | 31/03/09 | 5,26 | 8,52 |
| 329 | 30/04/09 | 6,51 | 9,38 |
| 330 | 29/05/09 | 3,51 | 5,31 |
| 331 | 30/06/09 | -1,76 | 0,01 |
| 332 | 31/07/09 | 12,93 | 7,40 |
| 333 | 31/08/09 | 0,56 | 3,34 |
| 334 | 30/09/09 | 1,31 | 3,56 |
| 335 | 30/10/09 | 0,83 | -1,98 |
| 336 | 30/11/09 | 5,21 | 5,74 |
| 337 | 31/12/09 | 3,59 | 1,77 |

| *Regression Statistics* | |
|---|---|
| Multiple R | 0,5418 |
| R Square | 0,2984 |
| Adjusted R Sq | 0,2968 |
| Standard Error | 6,1726 |
| Observations | 432 |

ANOVA

| | *df* | *SS* |
|---|---|---|
| Regression | 1 | 6968,2227 |
| Residual | 430 | 16383,6443 |
| Total | 431 | 23151,8672 |

| | *Coefficient* | *Standard Error* |
|---|---|---|
| Intercept | 0,2888 | 0,2989 |
| X Variable 1 | 0,9449 | 0,0699 |

# Regression Example: Output

- Estimate for alpha (monthly): $\alpha = 0.29\%$.
  - Standard error of estimate $s_\alpha = 0.31\%$.
- Estimate for beta: $\beta = 0.94$.
  - Standard error of estimate $s_\beta = 0.07$.
- Idiosyncratic volatility (sigma): $s(\epsilon) = 6.17\%$.
- R-Square: 29.8%.

# Beta

- Beta:

$$R_n - R_f = \alpha_n + \boxed{\beta_n}(R_M - R_f) + \epsilon_n.$$

  Measures the sensitivity of asset $n$ to market movements.

- If the return on the market portfolio is higher by 1%, then the return on asset $n$ is higher by $\beta_n$% (holding all else equal).
  - The beta of a stock is typically between 0.5 and 2.

- Beta is given by

$$\beta_n = \frac{Cov(R_n, R_M)}{V(R_M)} \simeq \frac{Cov(R_n - R_f, R_M - R_f)}{V(R_M - R_f)}.$$

# Alpha and Sigma

- Alpha:

$$R_n - R_f = \boxed{\alpha_n} + \beta_n(R_M - R_f) + \epsilon_n.$$

Measures the asset's attractiveness.

- Sigma:

$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \boxed{\epsilon_n}\,.$$

Sigma is the standard deviation of $\epsilon_n$, the asset's idiosyncratic risk.

Asset volatility depends on beta and sigma. Indeed, return variance can be decomposed as follows:

$$V(R_n - R_f) = \beta_n^2 V(R_M - R_f) + V(\epsilon_n).$$

# Expected (Excess) Return

- Regression equation

$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \epsilon_n.$$

- Taking expectations and using the fact that $E(\epsilon_n) = 0$, we get

$$E(R_n) - R_f = \alpha_n + \beta_n(E(R_M) - R_f).$$

- The expected excess return of asset $n$ depends on
  - the asset's alpha, $\alpha_n$
  - the asset's beta, $\beta_n$
  - the **market risk premium**, $E(R_M) - R_f$.

- The CAPM will yield some sharp prediction about alpha.

# 3. The CAPM

- The CAPM is a theoretical model which provides insight on assets' expected returns.
- Assumptions:
  - There are $N$ risky assets and a riskless asset.
  - Short sales are allowed and costless.
  - Investors care only about mean and variance.
  - Investors have the same beliefs.
  - Investors have a one-period horizon.

# Asset Demand

- We first consider the demand for assets.
- Recall Lecture 5: Any investor chooses a portfolio on the portfolio frontier (CML).
- That is, any investor holds a combination of the tangent portfolio and the riskless asset.
  - Very risk-averse: Portfolio closer to riskless asset.
  - Not very risk-averse: Portfolio closer to tangent portfolio, or even above tangent portfolio.
- Investors as a group:
  - Hold a combination of tangent portfolio and riskless asset.

# Supply: The Market Portfolio

- The **market portfolio** is the value-weighted portfolio of the $N$ risky assets.
- Let $P_n$ denote the price per share and $s_n$ the total number of shares. The market value (market capitalization) of asset $n$ is $P_n s_n$.
  - Total market capitalization is $\sum_{i=1}^{N} P_i s_i$.
- The weight of asset $n$ in the market portfolio is

$$w_n = \frac{P_n s_n}{\sum_{i=1}^{N} P_i s_i}.$$

- The return on the market portfolio is $R_M = \sum_{n=1}^{N} w_n R_n$.
- The 'market risk premium' is the expected excess return of the market portfolio:

$$E(R_M) - R_f = \left( \sum_{n=1}^{N} w_n E(R_n) \right) - R_f = \sum_{n=1}^{N} w_n \left( E(R_n) - R_f \right).$$

# Market Equilibrium

- Equilibrium: demand equals supply.
- Therefore, in equilibrium,

| The Tangent portfolio must coincide with the Market portfolio. |
|---|

- Example:
  - Suppose that GE has a weight of 0.7% in the tangent portfolio, whereas it has a weight of 1% in the market portfolio.
  - Supply of GE exceeds demand $\Rightarrow$ the price of GE has to fall.
  - As a result:
    - Weight of GE in market portfolio decreases.
    - Weight of GE in tangent portfolio increases (higher expected return).
  - The price falls until weights become equal.

# Towards the CAPM Equation

- Recall from Lecture 5 that any portfolio $P$ on the CML is such that the ratio

$$\frac{E(R_n) - R_f}{Cov(R_n, R_P)}$$

is independent of the particular asset $n$.

- In equilibrium, the market portfolio coincides with the tangent portfolio, which is on the CML.

- Therefore, the ratio

$$\frac{E(R_n) - R_f}{Cov(R_n, R_M)}$$

is equal across all assets $n$.

# Towards the CAPM (cont'd)

- This observation implies there exists some value $\lambda$ such that for all asset $n$

$$E(R_n) - R_f = \lambda Cov(R_n, R_M).$$

  An asset's expected return is proportional to its covariance with the market.

- A simple argument shows that (see proof next slide)

$$\lambda = \frac{E(R_M) - R_f}{V(R_M)}.$$

- Therefore, for any asset $n$

$$\begin{aligned} E(R_n) - R_f &= \frac{E(R_M) - R_f}{V(R_M)} Cov(R_n, R_M) \\ &= (E(R_M) - R_f) \times \frac{Cov(R_n, R_M)}{V(R_M)}. \end{aligned}$$

## Proof: The "Price of Covariance Risk", $\lambda$

- We know that there exists a value $\lambda$ such that for any asset $n$

$$E(R_n) - R_f = \lambda Cov(R_n, R_M).$$

- Premultiplying by $w_n$, the weight of asset $n$ in the market portfolio, and summing over all assets, we get

$$\sum_{n=1}^{N} w_n(E(R_n) - R_f) = \lambda \sum_{n=1}^{N} w_n Cov(R_n, R_M)$$

$$\Rightarrow \quad \sum_{n=1}^{N} w_n E(R_n) - \left(\sum_{n=1}^{N} w_n\right) R_f = \lambda Cov\left(\sum_{n=1}^{N} w_n R_n, R_M\right)$$

$$\Rightarrow \quad E(R_M) - R_f = \lambda Cov(R_M, R_M) \ = \ \lambda V(R_M).$$

- Therefore

$$\lambda = \frac{E(R_M) - R_f}{V(R_M)}.$$

# The CAPM Equation

- Using the definition of the beta of asset $n$

$$\beta_n = \frac{Cov(R_n, R_M)}{V(R_M)},$$

we obtain the main CAPM prediction:

$$\begin{aligned} E(R_n) - R_f &= (E(R_M) - R_f) \times \beta_n \\ i.e., \qquad E(R_n) &= R_f + \text{MRP} \times \beta_n. \end{aligned}$$

- Recall that the regression equation implies that

$$E(R_n) - R_f = \alpha_n + \beta_n(E(R_M) - R_f).$$

The CAPM predicts that $\alpha_n = 0$.

# 4. Key Insights from the CAPM

- The CAPM says

$$E(R_n) - R_f = (E(R_M) - R_f) \times \beta_n.$$

- An asset's expected return depends on the asset's risk
  - through the asset's beta (systematic risk),
  - and not through the asset's sigma (idiosyncratic risk).
- Key insight:

| |
|---|
| Systematic risk is "priced" in the market.<br>Idiosyncratic risk is not. |

In other words:

| |
|---|
| The relevant measure of asset risk<br>is beta and not the variance. |

# Intuition

- Suppose that an asset has **positive beta.** The CAPM implies that it has higher return than the riskless asset on average, $E(R_n) - R_f > 0$.
  - Intuition: The asset earns a higher expected return in compensation for the fact that it increases portfolio risk.

- Suppose that an asset has **zero beta.** The CAPM implies that its average return is equal to $R_f$, the rate of return on the riskless asset.
  - Intuition: The asset's risk is only idiosyncratic and can be diversified. The asset does not contribute to portfolio risk.

- Suppose that an asset has **negative beta.** The CAPM implies that it has lower return than the riskless asset on average.
  - Intuition: The asset reduces portfolio risk, i.e., provides insurance.

# Linearity

- The CAPM says

$$E(R_n) = R_f + (E(R_M) - R_f) \times \beta_n,$$

  and implies that an asset's expected return depends on risk only through beta.

- It also implies that the asset's expected excess return is linear in beta.

- For instance, if beta is 2, then the asset's expected excess return is twice the market risk premium.

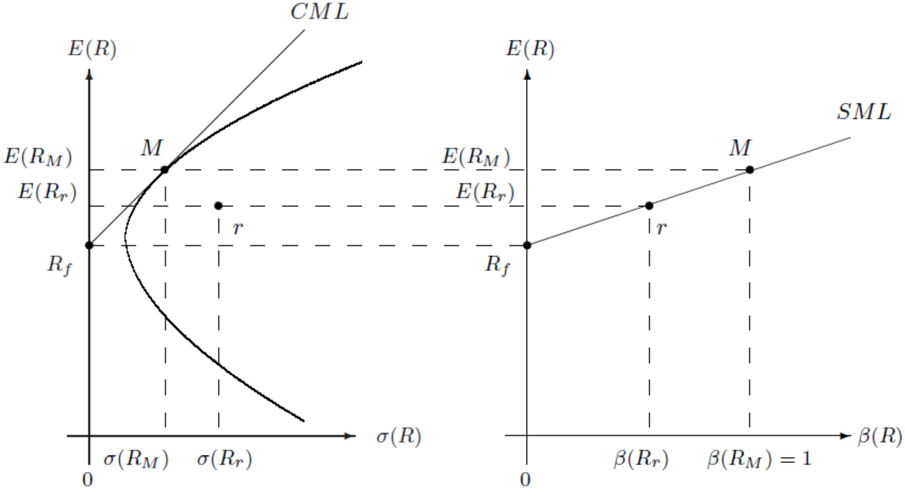
# 5. Summary: CML vs. SML

- The Capital Market Line **(CML)**
  - is in the standard deviation/expected return space.
  - contains all frontier portfolios (and only those).

  According to CAPM, the tangent portfolio coincides with the market portfolio ($T = M$).

- The Security Market Line **(SML)**
  - is in the beta/expected return space.
  - represents expected return as function of beta (according to the CAPM) for all securities and portfolios.

# CML vs. SML



$CML$
$E(R)$
$E(R_M)$
$M$
$E(R_r)$
$r$
$R_f$
$0$
$\sigma(R_M)$
$\sigma(R_r)$
$\sigma(R)$
$E(R)$
$SML$
$E(R_M)$
$M$
$E(R_r)$
$r$
$R_f$
$0$
$\beta(R_r)$
$\beta(R_M) = 1$
$\beta(R)$

# 6. Uses of the CAPM

- The CAPM provides a simple way to compute **risk-adjusted discount rate.**

- This is useful when we want to evaluate a stream of *risky* cashflows.
    - Valuation of stocks (Lecture 7).
    - Valuation of firms' investments (Corporate Finance).

- The CAPM also provides a useful **benchmark** to
    - Spot attractive investment opportunities.
    - Evaluate the performance of mutual fund managers.

  Key insight: return performance needs to be adjusted for beta risk.